An Application of Boolean Algebra to Switching Circuit Design

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This paper discusses the application of switching (Boolean) algebra to the development of an all-relay dial pulse counting and translating circuit employing the minimum number of relays. An attempt is made to outline what appears to be the most promising method of obtaining beneficial results from the use of the algebra in the design of practical switching circuits.

INTRODUCTION

The demands made upon telephone switching systems in regard to improvements in handling capacity, speed, flexibility and economy are continually increasing. In order to meet design objectives enabling the fulfillment of these demands, switching circuits have of necessity become more and more complex and intricate. As certain types of relay switching circuits increase in complexity, the problem of control and output contact network design becomes more and more laborious and time consuming. This is especially true in those circuits in which an attempt has been made to achieve the ultimate in efficiency and economy in that the number of relays used therein approaches the absolute minimum necessary to provide the required number of distinct output combinations. In this type of near-minimum combinational or sequential relay circuit there are numerous parallel control and output contact paths which thread through the same relays repeatedly, thereby causing the individual relay contact loads to become relatively large. Thus the designer's problem becomes that of first developing a workable control and output contact network and then manipulating and minimizing contacts within that network so that the maximum number of contacts used on any one relay is within that permissible on any commercially available relay having the necessary speed characteristics.

Even in those combinational and sequential relay circuits which are not near-minimum and therefore probably have fairly light individual relay contact loads, there are, of course, advantages to be gained by using the least number of contacts possible. Although the initial cost per additional contact (assuming that a few added contacts per relay will not impair the relay speed or space characteristics to an extent that the circuit requirements are not met) is almost negligible, there are other economic savings possible. Since each contact must be connected to the remainder of the contact network, minimizing contacts and consequently soldered connections means a saving in wiring time and labor. Furthermore, if the designer will manipulate the contacts so that the relays can be chosen from a comparatively few standardized codes, which are in large demand, it is possible to avoid the expensive stockpiling of numerous special designs having only a limited demand. In addition, using the least number of contacts minimizes the focal points of most relay circuit failures which are the contacts themselves (i.e., dirty or worn contacts).

It might also be noted at this point that electronic combinational or sequential circuits usually require electronic gating networks to perform functions which are completely analogous to those of relay contact networks. Hence, the same problem of minimization exists. However, in electronic circuits, gate minimization is even more advantageous since the cost per additional electronic gate is much higher than the cost per additional relay contact.

It is rather obvious that the multiplicity of paths in most combinational and sequential circuits can cause their design to become an extremely difficult and time consuming problem if the contact paths are developed with the aforementioned considerations in mind.

The circuit designer's usual approach to the solution of such contact minimization and manipulation problems is that of inspection. The method of inspection presupposes a background of considerable experience in that the designer must recognize certain contact network arrangements that may allow further rearrangements and thereby he must mentally develop his own rules. In order to check on any of his manipulations he must repeatedly redraw the network during this inspection design process. It is evident that this is often a long and tedious method and, depending on the skill of the designer, may or may not result in an optimum or even adequate solution.

Suitable contact network arrangements often appear only after consideration of several alternative schemes and the rearrangements of the network interconnections of these schemes. Realization of this makes it quite evident that any means of obtaining and comparing these various schemes quickly and with a mathematical accuracy which does not require continuous checking of network paths permits a more rapid and complete exploration of the particular problem. Switching algebra, first codified by C. E. Shannon[1], is the systematic application of G. Boole's[2]

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[1] C. E. Shannon, *A Symbolic Analysis of Relay and Switching Circuits*, Trans. AIEE, **57**, 1938.

[2] G. Boole, *The Mathematical Analysis of Logic* (Cambridge 1847) and *An Investigation of the Laws of Thought* (London 1854).

"Algebra of Logic" to switching circuits and is just such a means. It is a tool which can be used to investigate the complex combinational and sequential networks to determine satisfactory contact arrangements or reject unsatisfactory ones with a minimum of time and effort. It should be emphasized, however, that as with any tool, satisfactory results depend upon the judgment, ingenuity and logical reasoning of the user. Furthermore, as will be evident from the following development, switching (Boolean) algebra in its present state is not to be considered entirely selfsufficient but, for the most beneficial results, should be applied, when warranted, in conjunction with inspection techniques so that the latter may fill in any limitations in the algebra techniques which have not been completely systematized as yet due to the newness of this field.

The problem of solving the contact requirements of a minimum relay dial pulse counting and translating circuit recently developed as a component of the originating register of the No. 5 Crossbar System will be used as a means of illustrating the practical use now being made of switching algebra and of indicating exactly where the application of the algebra enters the design problem.

BASIC DIAL PULSE COUNTER REQUIREMENTS

The primary function of the originating register is to receive pulse signals representing digits from a telephone dial or similar calling device and to store a record of the digits in a form suitable for use by an external circuit. The dial pulse counting and translating circuit, an integral part of the originating register, is oriented with respect to other parts of the register by the block diagram of Fig. 1. The *L* relay is the pulse detecting relay. When the subscriber's switchhook contact is closed due to the lifting of the phone, the originating register is connected to the line and the *L* relay is operated. Thereafter it follows the breaks and makes of the subscriber's dial and feeds these repeated dial pulses into the counter. After the pulses are counted they are translated to a new code. In switching systems it is advantageous to translate from the basic dial ten pulse decimal code to a "two out of five" self-checking code. In this latter code any single error within the circuit will result in either one or three relays operated in the associated storage circuit rather than two and thus an error can readily be detected. The output of the translator is fed via a steering circuit to the register or storage circuit. The slow release *RA* relay is the pulse train detecting relay which holds between the individual pulses of a digit and releases only at the end of the pulse train. When it releases it activates the translating circuit and thereby transfers the translated code information to the storage circuit. The *RA1* relay in operating terminates the output from the translator and simultaneously releases the relays in the counter to prepare it for the next digit.

Specific requirements imposed by the originating register circuit necessitate the counting of one to eleven pulses; the use of a driving source consisting of a single break-make (or transfer) contact with ground on the armature spring; and outputs as follows:

1. Count of 1 through 10: ground on two of the 0, 1, 2, 4, 7 output leads in the combination corresponding to the count.
2. Count of 10: ground on the $Z0$ lead.
3. Count of 11: ground on the 0 lead only (this is a trouble-detecting feature).

In addition, the design of the steering and register-storage circuit requires that no output leads be connected together until the second pulse is received. Furthermore, each relay is limited to a combination of simple make and break contacts not exceeding a total of twelve. This utilizes the maximum number of springs obtainable on presently available relays and also avoids the larger armature gaps imposed by transfers which would result in a reduction in the relay speed of operation. Speed requirements also do not permit the use of shunt release in the circuit operation.

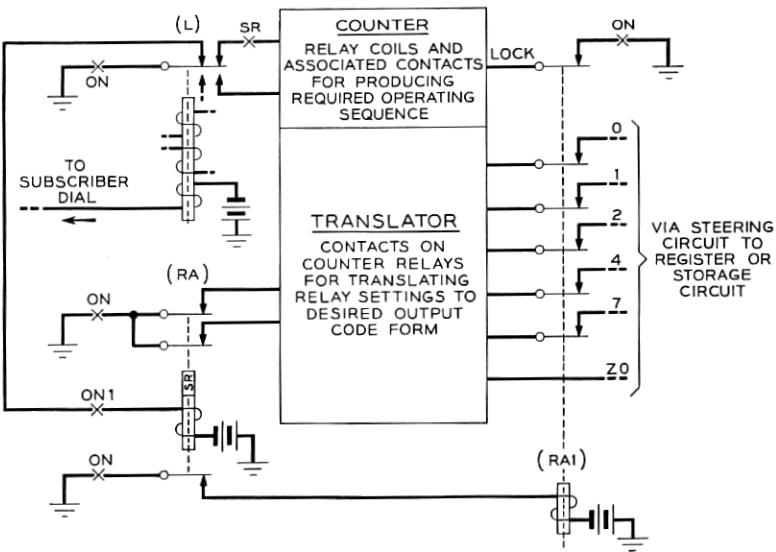


Fig. 1—The schematic of a portion of a dial pulse register circuit for counting decimal code pulses and translating them to "two out of five" signals. (In the symbolism used in the illustrations a cross indicates a "make" contact and a vertical bar indicates a "break" contact.)

THEORY OF A MINIMUM RELAY COUNTER

The counting circuit under consideration does not contemplate the use of any circuit elements other than relays that react to the beginning or end of a pulse. Therefore it must establish a distinct combination of relays operated or released during and between successive pulses. The minimum number of ordinary "two-position" relays, R, required to count P pulses can be obtained from the expressions (1) $2P \leq 2^R$ if the counter is to lock up during, or recycle after, the last pulse or (2) $2P \leq 2^R - 1$ if the counter is to lock up after the last pulse.

The usual counting circuit used for determining the number of pulses in a dial train is required to count ten pulses, however there are certain advantages in regard to trouble indications if the counter counts eleven pulses. In either case the minimum number of relays necessary, according to the preceding formulae, is five. It should be noted that the ease with which this minimum number can be attained depends upon whether the input is derived from a single, double or transfer contact source.

DETERMINATION OF OPERATING SEQUENCE

Having determined that the minimum number of relays necessary is five, the first step in design is to develop an operating sequence pattern from the resulting 2^5 or 32 possible relay combinations. These combinations may be utilized in any order deemed desirable to obtain the 23 distinct combinations needed to differentiate between eleven pulses (22 for the eleven makes and breaks plus an all-relays-normal combination). In this phase of the design switching algebra is not involved. The optimum sequence to meet a particular set of requirements can only be determined by repeated trials guided by an intimate knowledge of objectives.

Initial studies, made by Joseph Michal, of various possible sequence patterns for a five relay circuit, including those having a three relay "ring" followed by two auxiliary relays and those having a two relay pulse divider followed by three auxiliary relays, resulted in the conclusion that the latter approach was the most fruitful. The sequence pattern adopted is shown in detail in Table I. The pattern is extended through 12 pulses, and it can be seen that the nature of the sequence is such that this employs all 32 combinations of the 5 relays. Several of these are transient and occur during part of a pulse or inter-pulse interval. Examination of the tail end of the sequence indicates that it will be simpler to design on the basis of a full 12 pulses than attempt to block at the end of the 11 pulses specified by the requirements. If trouble con-

TABLE I

SEQUENCE OF OPERATION

	Pulsing Relay	Counting Relays					Relay Combination	Two out of Five Code
	L	A	B	C	D	E		
Seizure	0	1	1	1	1	1	1	
1st pulse	1	0	1	1	1	1	2	
	0	0	0	1	1	1	3	0,1
2nd pulse	1	1	0	1	1	1	4	
	1	1	0	0	1	1	5	
	0	1	1	0	1	1	6	0,2
3rd pulse	1	0	1	0	1	1	7	
	0	0	0	0	1	1	8	
	0	0	0	0	0	1	9	1,2
4th pulse	1	1	0	0	0	1	10	
	0	1	1	0	0	1	11	0,4
5th pulse	1	0	1	0	0	1	12	
	1	0	1	1	0	1	13	
	0	0	0	1	0	1	14	1,4
6th pulse	1	1	0	1	0	1	15	
	0	1	1	1	0	1	16	
	0	1	1	1	0	0	17	2,4
7th pulse	1	0	1	1	0	0	18	
	0	0	0	1	0	0	19	0,7
8th pulse	1	1	0	1	0	0	20	
	1	1	0	0	0	0	21	
	0	1	1	0	0	0	22	1,7
9th pulse	1	0	1	0	0	0	23	
	0	0	0	0	0	0	24	
	0	0	0	0	1	0	25	2,7
10th pulse	1	1	0	0	1	0	26	
	0	1	1	0	1	0	27	4, 7-ZO
11th pulse	1	0	1	0	1	0	28	
	1	0	1	1	1	0	29	
	0	0	0	1	1	0	30	0
12th pulse	1	1	0	1	1	0	31	
	0	1	1	1	1	0	32	0

Total of $2^5 = 32$ combinations used.

Note: 0 is used to indicate that the relay listed at the head of the column is operated, and 1 is used to indicate that the relay is released.

ditions introduce pulses beyond 12, the circuit will without difficulty recycle through combinations corresponding to pulses 11 and 12.

Table I also indicates the leads which must be grounded in order to provide the translations to the "two out of five" and "single lead" codes.

The characteristics of this circuit may be summarized as follows: It contains only five relays which is the absolute minimum necessary. It uses all 32 of its available combinations. Its control and translating job is complex enough to indicate the need for a considerable number of contacts and hence the need for extensive contact manipulation to minimize and distribute these contacts.

It is apparent that a great deal of time would be necessary to accomplish this manipulation by inspection methods, thereby indicating the need for an additional tool such as switching algebra to assist the designers in this task.

ALGEBRAIC METHODS APPLIED TO CONTROL CIRCUIT

The sequence of operations of Table I is used as the starting point in the application of the algebra. The exact calculations necessary to develop the control and translating circuit by this means are shown in detail later. However, the individual steps in the solution might well be outlined here. First, the design of the control and translating networks will be regarded as separate problems. In theory these can be integrated together, but the resultant network is likely to be so complex that understanding and maintenance of the circuit would suffer. Each of the two networks can be individually considered as a multi-terminal network of the single input type. That is, the control network is an associated set of contacts which connects a single ground input to the windings of five relays, and the translating network is an associated set of contacts which connects a single ground input to the six output leads. Since switching algebra is directly applicable to two-terminal networks rather than multi-terminal networks, the approach to this particular problem is of necessity somewhat indirect.

The most satisfactory method of attack is to develop first a two-terminal network for each of the output paths of the multi-terminal network under consideration. The two-terminal networks can be expressed algebraically and manipulated into their simplest form by means of the switching algebra theorems to be given later. The individual networks can then be inspected carefully, either in algebraic or circuit form, with the objective of combining them in the most advantageous fashion. It will be found, in general, that the simplest network configurations do not readily combine and that further manipulation is necessary to obtain an economical circuit. It is at this point that the algebra achieves its greatest utility, since its application permits the simple and rapid changing of a given two-terminal network into a large variety of different forms with mathematical assurance that circuit equivalence is maintained. Inspection of the networks in the several forms provides clues as to the preferable combining forms and often indicates additional manipulations that might be desirable.

This network development is a combination of mathematics and integration by inspection. It is characterized by repeated trials of alternative forms and at no stage is there any definite assurance that the optimum circuit has been attained. However, the ease of manipulation provided by the algebra greatly enhances the probability of designing a better circuit than would be possible by inspection alone. In combining the two-terminal networks, care must be taken not to introduce "sneak" paths which improperly connect outputs together. The algebra usually offers means of introducing one or two additional contacts which permit combining networks and yet eliminate the adverse effects of the sneak paths.

The above procedure will now be carried out in detail with the switching algebra theorems that are used in all the following algebraic manipulations noted at the margin by the number which corresponds to the number of the theorem in the complete listing in Table II. This table is

TABLE II
SWITCHING (BOOLEAN) ALGEBRA*

Definitions	Postulates
Addition	(1) $X = 0$ or $X = 1$, where X is a contact or a network.
$(+) = AND =$ Series	(2a) $0 \cdot 0 = 0$
Multiplication	(2b) $1 + 1 = 1$
$(\cdot) = OR =$ Parallel	(3a) $1 \cdot 1 = 1$
Circuit States	(3b) $0 + 0 = 0$
$0 =$ Closed Circuit	(4a) $1 \cdot 0 = 0 \cdot 1 = 0$
$1 =$ Open Circuit	(4b) $0 + 1 = 1 + 0 = 1$

Theorems

(1a) $X + Y = Y + X$	(8a) $X' + X = 1$
(1b) $XY = YX$	(8b) $X'X = 0$
(2a) $X + Y + Z = (X + Y) + Z = X + (Y + Z)$	(9a) $0 + X = X$
(2b) $XYZ = (XY)Z = X(YZ)$	(9b) $1 \cdot X = X$
(3a) $XY + XZ = X(Y + Z)$	(10a) $1 + X = 1$
(3b) $(X + Y)(X + Z) = X + YZ$	(10b) $0 \cdot X = 0$
(4a) $X + X = X$	(11a) $(X + Y')Y = XY$
(4b) $XX = X$	(11b) $XY' + Y = X + Y$
(5a) $X + XY = X$	(12a) $(X + Y)(X' + Z)(Y + Z) = (X + Y)(X' + Z)$
(5b) $X(X + Y) = X$	(12b) $XZ + X'Y + YZ = XZ + X'Y$
(6a) $(X)' = X'$	(13) $(X + Y)(X' + Z) = XZ + X'Y$
(6b) $(X')' = X$	(14a) $f(X) = A \cdot f(X)_{A=1,\, A'=0} + A' \cdot f(X)_{A=0,\, A'=1}$
(7a) $(X + Y + Z + \cdots)' = X' \cdot Y' \cdot Z' \cdot \cdots$	(14b) $f(X) = [A + f(X)_{A=0,\, A'=1}][A' + f(X)_{A=1,\, A'=0}]$
(7b) $(X \cdot Y \cdot Z \cdot \cdots)' = X' + Y' + Z' + \cdots$	

* Reprinted from *The Design of Switching Circuits* by Keister, Ritchie and Washburn with the permission of D. Van Nostrand Co., Inc.

taken from *The Design of Switching Circuits* by Keister, Ritchie and Washburn*. The development of the algebraic expressions from the sequence of operations table will be in exact parallel to the methods suggested in the aforementioned text.

The symbolism adopted in the following development is basically that of using the notation A for all the make contacts on the A relay, and A' for all the break contacts on the A relay. Contacts or groups of contacts in series are related by the symbol of addition $(+)$ and contacts or groups of contacts in parallel are related by the symbol for multiplication $(\cdot)$ which may or may not be explicitly written, as in ordinary algebra. Therefore $(A + B')$ symbolizes a series contact path that is closed when the A relay is operated *and* the B relay is released, while (AB') symbolizes the parallel contact path that is closed when either A is operated *or* B is released. Switching algebra includes only two numerical values, 0 and 1, with the quantity 0 assigned to represent a closed path and 1 to represent an open path. For the tabular notation of Table I, 0 is used to indicate that the relay listed at the head of the column is operated and 1 is used to indicate that the relay is released.

As stated earlier, the present application of switching algebra utilizes the sequence of operation chart of Table I. The operate and release combinations for controlling the A, B, C, D and E relays can be selected from this table by observing where each relay to be controlled changes state. For example, the operate combination for relay D is relay combination 8 and the release combination for relay D is relay combination 24. It is not necessary to include the contacts of a relay in its own operate and release combinations. Note that the A and B relays which serve as a pulse divider can be controlled solely by the L relay and contacts on A and B without reference to C, D, E. However the C, D and E relays are internally controlled by all five counting relays. The development of all these control paths uses the following abbreviations:

$$g(X) = \text{operating combinations for the } X \text{ relay}$$

$$r(X) = \text{releasing combinations for the } X \text{ relay}$$

$$h(X) = \text{holding combinations for the } X \text{ relay}$$

$$X = \text{make contact on the } X \text{ relay}$$

Furthermore as expressed by theorem (6a and 6b) the negative of a contact network X is defined as a network which is a closed path under all conditions for which X is open, and is open under those conditions

* D. Van Nostrand, 1951. The Bell Telephone Laboratories Series.

for which X is closed. Hence $h(X)$ may be obtained from $r(X)$ by noting that $h(X)$ is the negative of $r(X)$. Therefore the entire control path of any relay can be expressed generally as

$$f(X) = g(X)[X + h(X)] = g(X)[X + (r(X))']$$

Thus for the A relay

$$g(A) = L' + B'$$

$$r(A) = L' + B$$

$$h(A) = [L' + B]' = LB' \qquad (7a)$$

and

$$f(A) = (L' + B')(A + LB')$$

$$= (L' + B')(A + L)(A + B') \qquad (3b)$$

$$= (L + A)(L' + B') \qquad (12a)$$

Also for the B relay

$$g(B) = L + A$$

$$r(B) = L + A'$$

$$h(B) = [L + A']' = L'A \qquad (7a)$$

and

$$f(B) = (L + A)(B + L'A)$$

$$= (L + A)(B + L')(B + A) \qquad (3b)$$

$$= (L + A)(L' + B) \qquad (12a)$$

The schematic forms of the A and B control circuits as represented by the above algebraic expressions are shown in Fig. 2a and 2b. Since the general requirements of the basic problem specify only one transfer on the L relay, only simple makes and breaks on the A and B relays and no shunt release paths (to avoid reduction in speed of operation), the combination of the above specific circuits is not possible without recourse to double windings. Another factor which affects the practical form of the circuit is the finite transit time of the L relay armature spring. Switching algebra presupposes instantaneous action of relay contacts and in certain cases, when the use of a break-make transfer is required, additional contacts are necessary to cover the open contact interval. The final circuit form, conceived by F. K. Low, is shown on Fig. 2c and uses an added A

contact. Algebraic equivalence of this circuit with the original is shown below.

$$\begin{aligned} f(A) &= (L'A + B')(L + A) \quad \text{(Fig. 2c)} \\ &= (L' + B')(A + B')(L + A) && \text{(3b)} \\ &= (L' + B')(L + A) && \text{(12a)} \\ f(B) &= (L'A + B)(L + A) \quad \text{(Fig. 2c)} \\ &= (L' + B)(A + B)(L + A) && \text{(3b)} \\ &= (L' + B)(L + A) && \text{(12a)} \end{aligned}$$

For the C relay which operates and releases twice in the entire 32 combination cycle

$$\begin{aligned} g(C) &= (A' + B + D' + E')(A' + B + D + E) \\ r(C) &= [(A + B' + D + E')(A + B' + D' + E)] \\ h(C) &= [(A + B' + D + E')(A + B' + D' + E)]' \\ &= (A'BD'E + A'BDE') && \text{(7a, 7b)} \end{aligned}$$

and

$$\begin{aligned} f(C) &= [(A' + B + D' + E')(A' + B + D + E)] \\ &\qquad [C + A'BD'E + A'BDE'] \\ &= [A' + B + (D' + E')(D + E)] \\ &\qquad [C + A'B(D' + E')(D + E)] && \text{(3a, 3b, 13)} \\ &= [A' + B + (D' + E')(D + E)][C + A'B] \\ &\qquad [C + (D' + E')(D + E)] && \text{(3b)} \\ &= [(A' + B)C + (D' + E')(D + E)][C + A'B] && \text{(3b)} \end{aligned}$$

The schematic circuit which the above represents is shown in Fig. 3a. Circuits of this type which use certain contacts more than once can some-

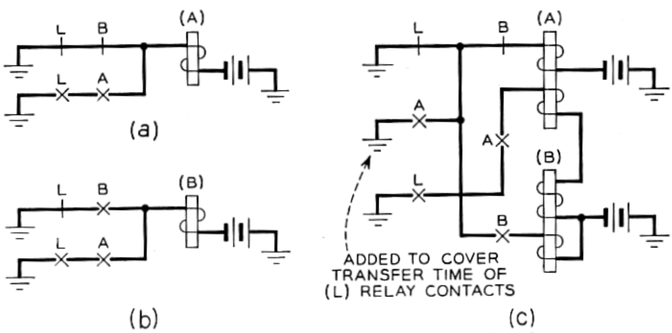


Fig. 2—Pulse divider of counting circuit.

times be drawn in bridge form with a consequent saving of contacts. One method is to manipulate the expression into a form which is known to be the series-parallel equivalent of a bridge. However, following usual algebraic procedures it is often difficult to recognize where this is possible. In the present case a method developed by G. R. Frost (not yet published) was used effectively. This resulted in the bridge circuit of Fig. 3b which has the series-parallel equivalent:

$$f(C) = [C + (D' + E')(D + E)][A' + B + (D' + E')(D + E)][C + A'][C + B]$$

By use of theorem (3b) this is seen to be equivalent to the previous expression for $f(C)$.

For the D relay which only operates and releases once in the entire cycle

$$\begin{aligned} g(D) &= (A + B + C + E') \\ r(D) &= (A + B + C + E) \\ h(D) &= (A + B + C + E)' \\ &= A'B'C'E' \end{aligned}$$

and $f(D) = (A + B + C + E')(D + A'B'C'E')$.

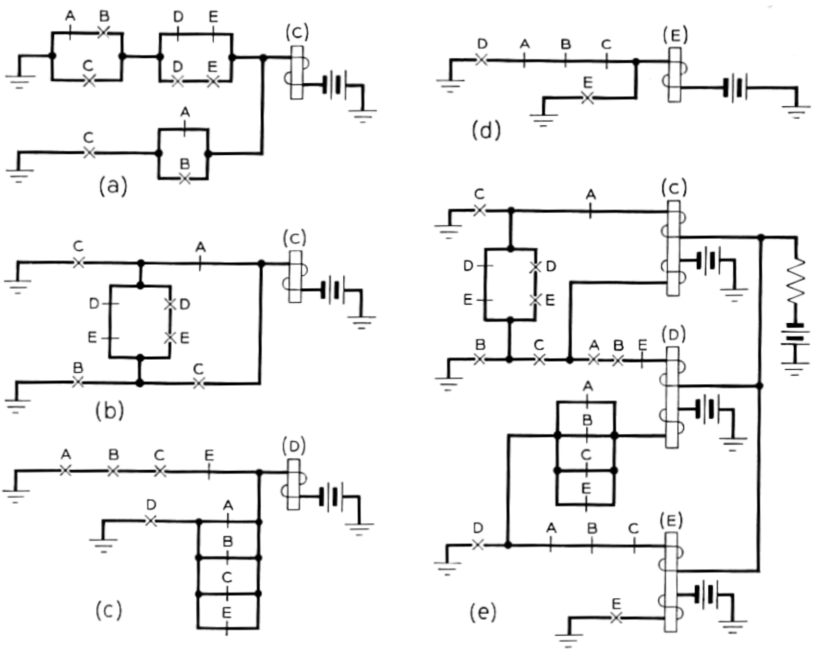


Fig. 3—Internal control of counting circuit.

By noting that $A + B + C$ is the negative of $A'B'C'$, this can be reduced to

$$f(D) = (A + B + C + E')(D + A'B'C') \qquad (3b, 12a)$$

However, this transformation introduces a hazard caused by the transit time of A relay contacts in passing from relay combination 9 to 10. Therefore the original expression will be used for relay D. The control path is shown in Fig. 3c.

For the E relay which operates and locks only once in the cycle

$$g(E) = (A' + B' + C' + D)$$

$$h(E) = E$$

and

$$f(E) = (A' + B' + C' + D)(E + E)$$

$$(A' + B' + C' + D)E \qquad (4a)$$

This control path is shown in Fig. 3d.

Apart from the problem of developing the required contact network, the practical problem of what operating power must be given to the relays in order to meet speed requirements must be dealt with. Since the use of low resistance windings in series with protective external resistors is called for to obtain the speed required, it appears that the use of two windings per relay might prove advantageous. By operating on the low resistance winding while locking on the high resistance winding, the current drain may be reduced (thereby saving a fuse) and furthermore some code reduction may be made possible as shown later. If double windings are used, two of the external series resistors may be eliminated by combining the control network so as to make certain that only one of the low resistance windings on the C, D, or E relays is energized at any one time. This would permit the use of one common external resistance with the aforementioned relays instead of three.

Keeping these practical considerations in mind, further savings may be made by combining the control circuits as shown in Fig. 3e. Although there is in this circuit a possibility of contact stagger on the A relay contacts causing the C and D low resistance windings to be energized at the same time, this will not be harmful since, when the stagger occurs, both relays are firmly locked operated by their high resistance holding windings.

TRANSLATING CIRCUIT

The translating circuit is particularly adaptable to switching algebra manipulation. Table I shows the combinations which prevail at the end of each pulse and the necessary "code" leads that must be grounded at these times. Reference to the block diagram of Fig. 1 shows that the output of the translator is not activated until the slow release *RA* relay releases after the last pulse of a digit has been received. Therefore the *A* relay can be eliminated from these combinations since at the end of every pulse the *A* and *B* relays are either both operated or both released and hence only one is needed to indicate the condition of both. Table III lists the numerous combinations which must close a ground path through to each of the five code leads and the *Z*0 lead. At the conclusion of the algebraic manipulation, the *A* and *B* contacts may be redistributed evenly since they perform interchangeable functions in translation.

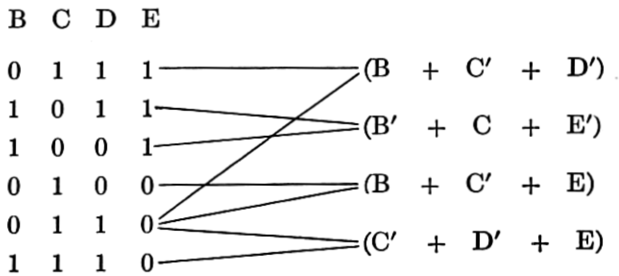
The objective in the design of the translating circuit is to obtain the most economical contact network subject to a spring distribution that

TABLE III
TRANSLATION

Output Lead Grounded	Counting Relays				Decimal Pulse
	B	*C*	*D*	*E*	
0	0	1	1	1	1
	1	0	1	1	2
	1	0	0	1	4
	0	1	0	0	7
	0	1	1	0	11
	1	1	1	0	12
1	0	1	1	1	1
	0	0	0	1	3
	0	1	0	1	5
	1	0	0	0	8
2	1	0	1	1	2
	0	0	0	1	3
	1	1	0	0	6
	0	0	1	0	9
4	1	0	0	1	4
	0	1	0	1	5
	1	1	0	0	6
	1	0	1	0	10
7	0	1	0	0	7
	1	0	0	0	8
	0	0	1	0	9
	1	0	1	0	10
*Z*0	1	0	1	0	10
Inconsequential combinations that may be used for simplification					
	0	0	1	1	
	1	1	0	1	
	0	0	0	0	
	1	1	1	1	

fits in with the control contact network. Again, this is a multi-terminal network problem and the procedure is to design two-terminal networks that combine most readily. Since it is impractical to illustrate all the repeated trials that led to the final design, each network will be designed separately with the understanding that some of the steps are imposed by the form of all networks viewed collectively.

The procedure adopted for developing the "0" lead network is as follows. First set up the miniature table repeating the portion of Table III that corresponds to the "0" lead. These parallel combinations should then be manipulated algebraically to obtain the greatest simplification possible. It is rather easy to apply some of the algebraic rules by observing the condition of the relay in the several combinations in the table. A simple "shorthand" rule to follow is: if in the table of combinations describing a particular two terminal network, all possible combinations of certain relays appear in conjunction with a single combination of other relays, the network contacts on the former relays may be neglected. In other words when 2^n different combinations of any number of variables m, are identical in all but n columns, contacts on the corresponding n relays are not required. This procedure is carried out below.



Thus we have the following algebraic expression for the "0" lead, which can be simplified as shown.

$$(B + C' + D')(B' + C + E')(B + C' + E)(C' + D' + E)$$

$$[C' + (B + D')(B + E)(D' + E)](B' + C + E') \qquad (3b)$$

$$[C' + (E + BD')(B + D')](B' + C + E') \qquad (3b)$$

This is shown on Fig. 4a. A somewhat different manipulation of the equation permits placing the network in the bridge form of Fig. 4b. The algebraic equation, given below, can easily be shown to be the equivalent of the original.

$$[E + C' + B(B' + D')](B' + C + E')(B + C' + D')$$

In certain cases the use of theorem (14b), normally employed to reduce the contacts of a particular relay to a single make and break, can produce simplifications difficult to accomplish otherwise. This is shown below, with the theorem applied with respect to relay E since E tended otherwise to be heavily loaded.

$$(B + C' + D')(B' + C + E')(B + C' + E)(C' + D' + E)$$

$$[E + (B + C' + D')(B' + C + 1)(B + C' + 0)(C' + D' + 0)]$$
$$[E' + (B + C' + D')(B' + C + 0)(B + C' + 1)(C' + D' + 1)] \quad (14b)$$

$$(E + C' + BD')[E' + (B' + C)(B + C' + D')] \quad (9a, 10a, 3b, 9b, 5b)$$

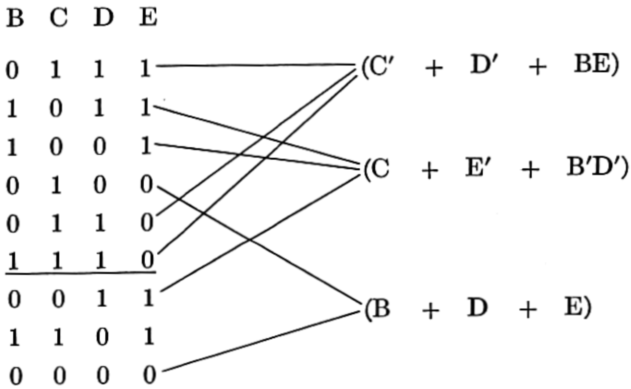
By modifying the first factor of the final expression in accordance with theorem 11a, this equation can be put in bridge form as shown on Fig. 4c.

$$[E + C' + B(B' + D')][E' + (B' + C)(B + C' + D')]$$

The above equation uses the same contacts as the previous expression, and although the right hand member is in a slightly different form, the expression is equivalent to the one obtained earlier.

When it is known that output conditions are inconsequential for some relay combinations, these inconsequential relay combinations may be combined with valid combinations to eliminate contacts in the network. Inconsequential means that the output during these particular combinations does not affect the proper functioning of the circuit. Four such combinations are listed in Table III. Only those inconsequential combinations which will combine readily with the actual combinations, thereby resulting in a reduction in the number of contacts, are to be used. Although the use of all the all-relays-released condition may be helpful in certain cases, it will not be used in the circuit under consideration since its use makes the requirement that no tie shall exist between output leads until the second pulse is received hard to meet.

With this in mind the "0" lead network is again examined. Note the use of another "shorthand" rule which states that if a part of the 2^n possible combinations is used in closing a path, the negative of the unused part of the 2^n possible combinations is equivalent to the original combinations. Thus if in the case at hand three of the possible four combinations of the B and C relays occur in series with the same combination of the D and E relays, the expression used is that for the series path of the D and E relays plus the negative of the missing combination of the B and C relays. In the following tabulation the combinations below the horizontal line are inconsequential.



The expression becomes:

$$(C + E' + B'D')(C' + D' + BE)(B + D + E)$$

$$[E + (C + 1 + B'D')(C' + D' + B0)(B + D + 0)]$$
$$[E' + (C + 0 + B'D')(C' + D' + B1)(B + D + 1)]$$
(14b on E)

$$[E + (C' + D')(B + D)][E' + (C + B'D')(C' + D' + B)]$$
(9a, 9b, 10a, 10b)

$$[E + (C' + D')(B + D)][E' + CD' + CB + B'D'] \qquad \text{(8b, 9a, 4b, 5a)}$$

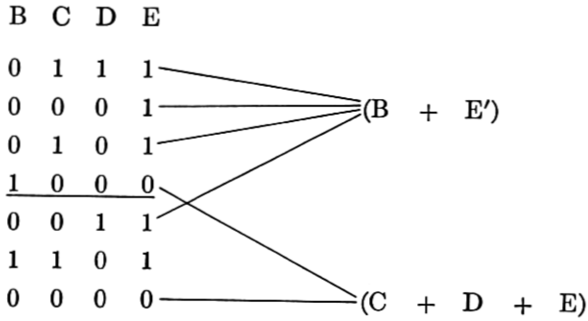
$$[E + (C' + D')(B + D)][E' + BC + B'D'] \qquad \text{(12b)}$$

$$[E + (C' + D')(B + D)][E' + (B + D')(B' + C)] \qquad \text{(13)}$$

Fig. 4d shows the schematic of the above expression. It is possible to put this in a bridge form without other changes because of the manner in which the front and back contacts of D are related to the other contacts. Comparison of all the circuits of Fig. 4 indicates that they all use the same number of contacts although final decision should be postponed until all the output circuits are obtained and the ease of combination of the different circuits can be compared.

The procedure for determining the remaining code leads is carried out on the following pages.

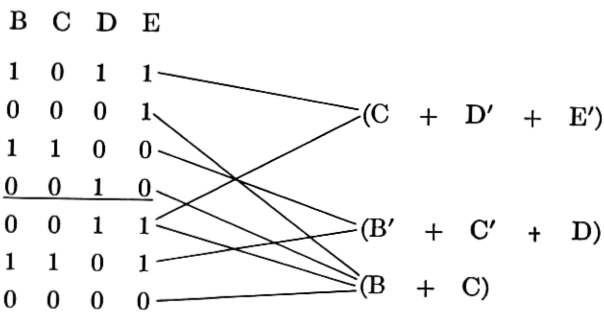
"1" lead—



resulting in $[E + C + D][E' + B]$ which is shown on Fig. 5a. It will later be found advantageous, in combining, to include the B' term in the first factor, giving the expression:

$$(E + B' + C + D)(E' + B) \quad \text{shown on Fig. 5b}$$

"2" lead—



hence

$$(C + D' + E')(B' + C' + D)(B + C)$$

or

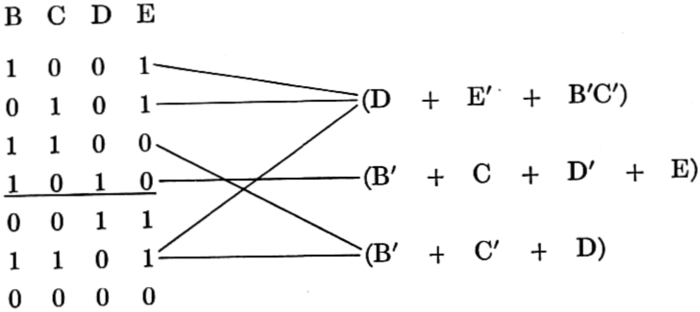
$$[C + B(D' + E')][C' + B' + D] \tag{3b}$$

For later ease in combining, this is changed to:

$$[C + B(B' + D' + E')][C' + B' + D] \tag{11a}$$

shown on Fig. 5c.

"4" lead—



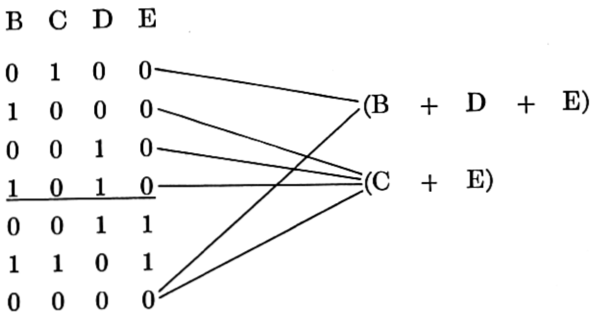
hence

$$(D + E' + B'C')(B' + C' + D)(B' + C + D' + E)$$

$$[D + (B' + C')(B'C' + E')][D' + B' + C + E] \qquad (3b)$$

which is shown on Fig. 5d.

"7" lead—



hence

$$(B + D + E)(C + E)$$

or

$$E + C(B + D) \qquad (3b)$$

which is shown on Fig. 5e.

"ZO" lead—

B	C	D	E
1	0	1	0

hence one has $(B' + C + D' + E)$ which is shown in Fig. 5f.

The final contact savings are achieved by combining the various output paths. The combined translation circuit that appears to be as reduced as possible is shown in Fig. 6. Note that certain forms of the individual output paths combine more readily than others. For example Fig. 4c and 5b combine more readily with the remaining paths than Fig. 4b and 5a. Note also that sometimes it is not the most reduced form of the individual output paths that permits efficient combining. This is

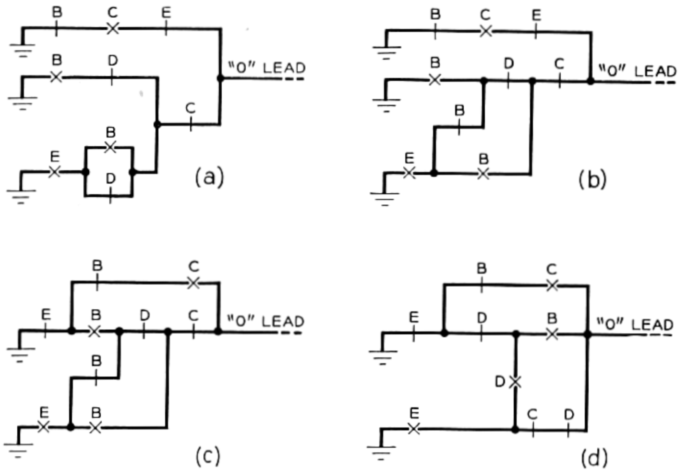


Fig. 4—The "0" lead of the translating circuit.

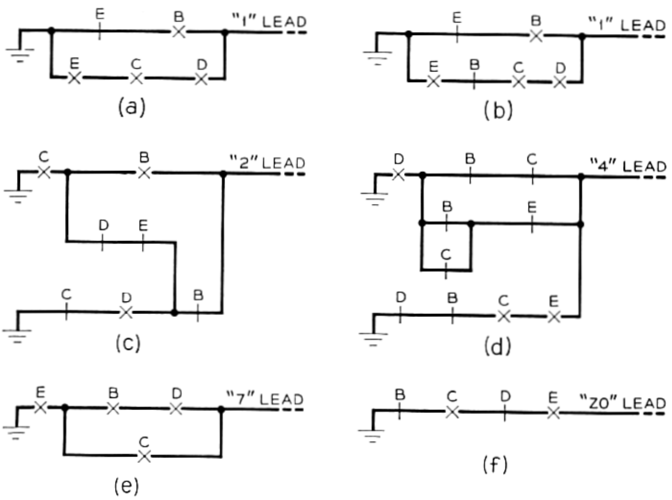


Fig. 5—The "1, 2, 4, 7, and Z0" leads of the translating circuit.

exemplified by the use of Fig. 5b rather than Fig. 5a. Although various forms of all of the output leads were tested for efficient combination only the form used is shown for outputs other than the "0" and "1" leads.

It is essential to scrutinize the final network for possible sneak paths. Sometimes to avoid these sneak paths it is necessary to add one contact on one relay to allow savings on others. Here again the inspection techniques go hand in hand with switching algebra and the need for both is obvious. The algebra obtains the various forms which are capable of

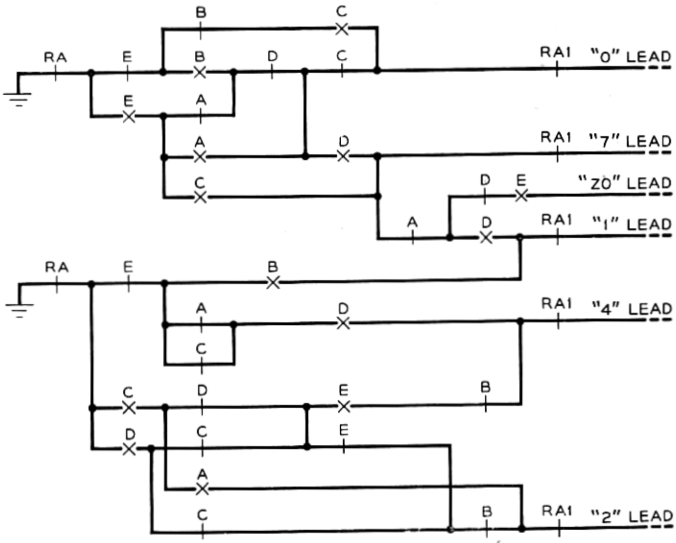


Fig. 6—Combined translating circuit.

different degrees of combination very quickly and efficiently. The inspection method is then necessary for the actual combination of these forms.

The additional *RA* relay contact is necessary to assist in avoiding interconnections between the output leads until after the second pulse is received. The final assignments of either *A* or *B* relay contacts are chosen to equalize the load on these relays.

THE COMPLETED CIRCUIT

The final form of the counting and translating circuit is shown on Fig. 7. The relays are all double wound to gain the benefits of current drain reduction. One additional advantage of using double windings is the relay code reduction made possible since now only two codes are necessary. One code serves the *A* relay and one other code serves the

B, C, D and E relays. In comparison to this total of five relays and two codes the circuit in present use in the latest crossbar system requires ten relays and seven codes.

AN ALTERNATE DESIGN OF THE PULSE DIVIDER

To illustrate the application of algebra where the apparatus contemplated puts less premium on contact minimization but more on standardization and winding minimization, certain modifications of the proposed circuit are considered.

In the event that new apparatus developments make possible the construction of relays that meet the necessary speed requirements even though winding impedance is increased, it appears possible (if the pulse divider is redesigned) to use only one code having a single winding for all five relays. The use of added contacts might be allowable if the new type of relay carries more springs than the present relay.

The redesign of the pulse divider to use single windings can be accomplished by manipulation of the basic algebraic expressions derived earlier for the pulse divider.

Thus for the A relay

$$\begin{aligned} f(A) &= (L' + B')(A + LB') \\ &= [L' + B'][A + B'(L + B)] \qquad (11a) \end{aligned}$$

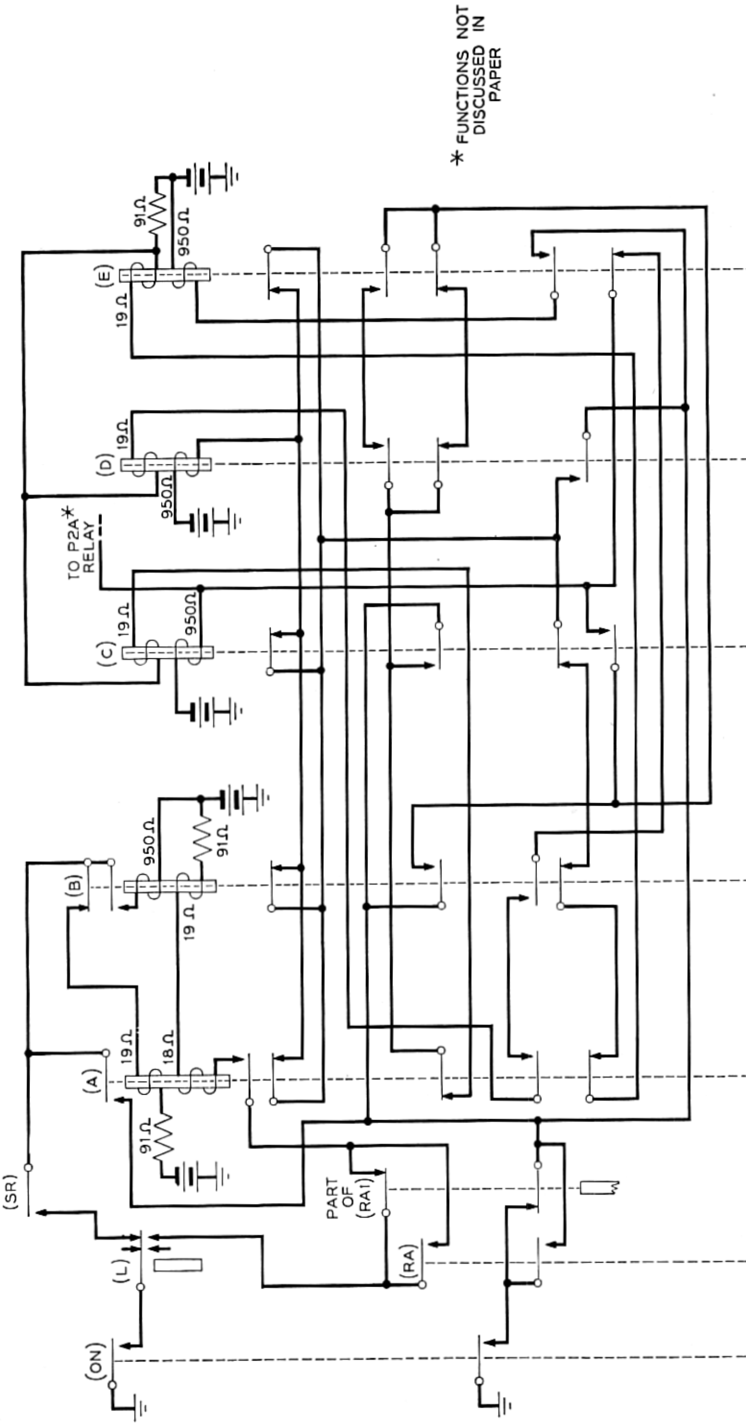
By attempting to manipulate the B relay control circuit into the same form, one obtains

$$\begin{aligned} f(B) &= (L + A)(B + L'A) \\ &= (L + A)(L' + B)(A + B) \qquad (3b) \\ &= [L' + B][A + BL] \qquad (3b) \\ &= [L' + B][A + B(L + B')] \qquad (11a) \end{aligned}$$

The schematics represented by the above algebraic expressions are shown in Fig. 8a and 8b. The circuit of Fig. 8c is obtained by combining the first two circuits so that only a single transfer is needed on the L relay. Note however that it is necessary to make the lower two B transfers have continuity action to insure proper functioning. Fig. 8d shows the pulse divider drawn in conventional form.

CONCLUSION

As far as is known, the dial pulse counting and translating circuit described herein requires fewer relays than any other circuit with similar



CONTROL CIRCUIT

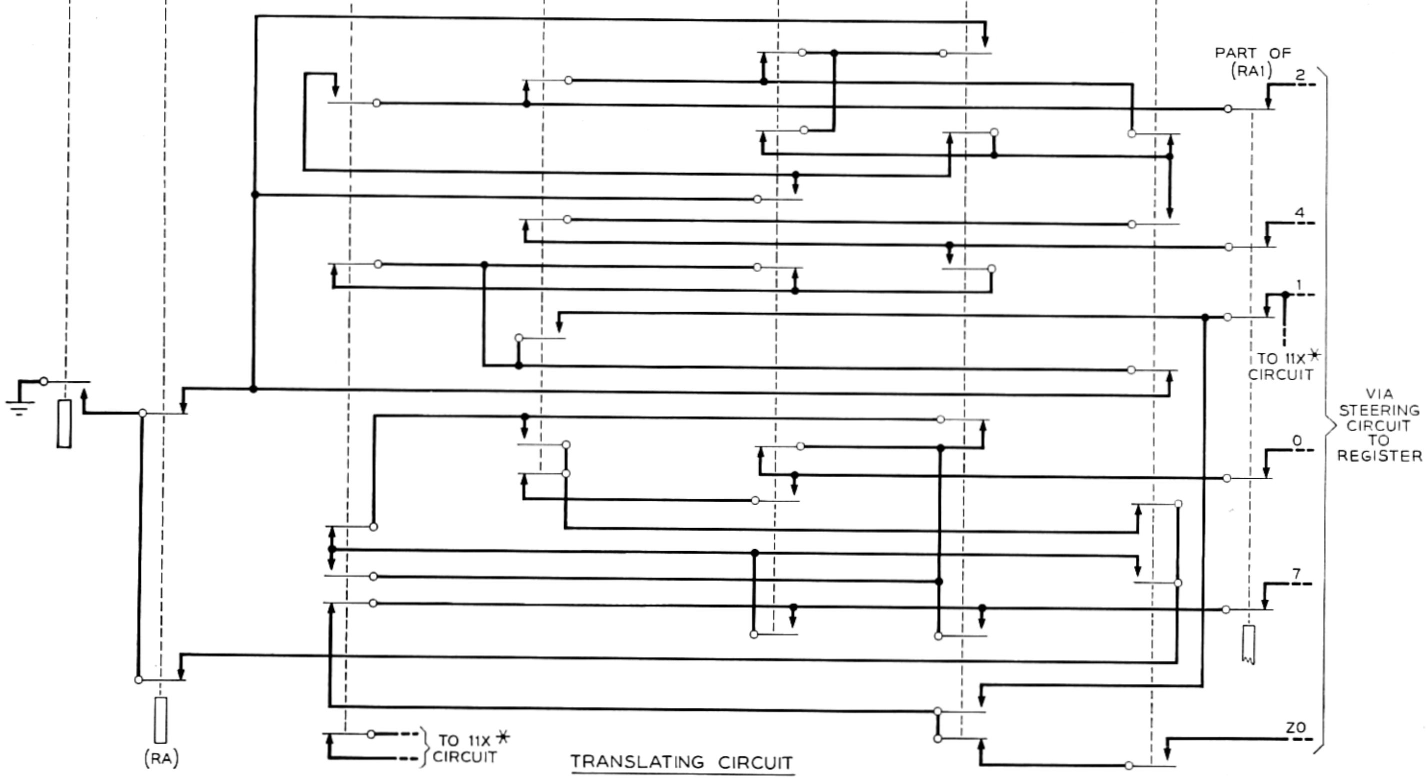


Fig. 7—The complete all-relay dial-pulse counting and translating circuit employing the minimum number of relays.

functions at present employed in Bell System standard switching equipment. The previous dial pulse counter used in the latest crossbar system required a total of ten relays. Thus the present design represents a considerable saving in cost and space. To a certain extent this result can be ascribed to the use of switching algebra during the circuit development.

Relay circuits designed on the basis of utilizing a large proportion of the possible combinations permitted by the component relays usually require heavy spring pile-ups. Since general purpose relays are limited in the number of springs which they can carry, this type of circuit usually

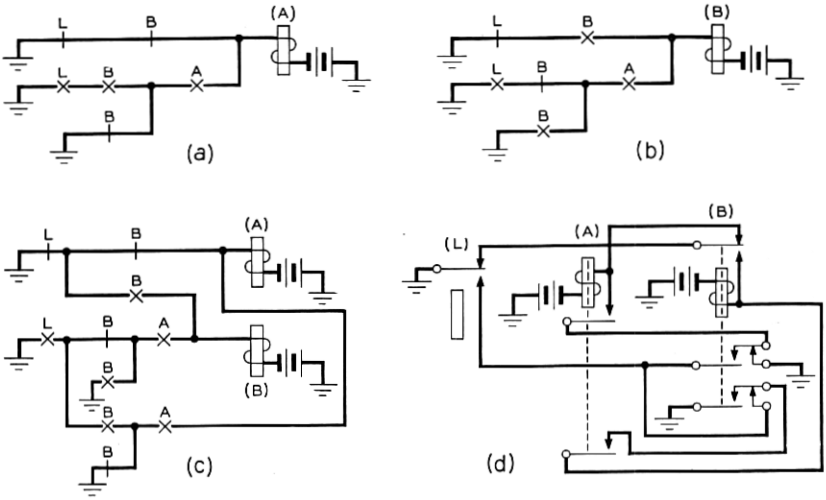


Fig. 8

entails considerable design effort to make most effective use of the available springs. Application of switching algebra to this aspect of the design problem can often provide crucial assistance.

It is recognized that switching algebra, in its present state of development, does not permit complete mathematical statement and manipulation of multi-terminal networks as represented by the counting and translating circuits. It does provide, however, facilities in manipulating two-terminal networks into a variety of forms from which can be selected those that combine most readily. This can result not only in a saving of time, but also in improved circuits which might not be realized by other design techniques. Unfortunately the algebra in its present state does not indicate when the optimum circuit has been attained. To some extent this is caused by apparatus or circuit considerations to which, since it is concerned solely with contact networks, the algebra does not apply. Thus, there is still considerable room left for the ingenuity and judgment of the switching circuit designer.

As a result of the experience in designing the dial pulse counter and translator, certain observations on the use of the algebra are believed to be valid. Although switching algebra may be used in the design of the simplest circuits, the most noticeable benefits are obtained by the application of the algebra to the design of those circuits in which the control and output paths are complex and interrelated. The particular minimum relay counting and translating circuit under discussion is an excellent example of this type of circuit. A secondary advantage of the algebra is its compact notation and its value as an efficient circuit "bookkeeping" method.

ACKNOWLEDGMENTS

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